[2] $\lim_{n\to\infty}\left|\dfrac{4^{n+1}\left(\tan\frac{2}{n+1}\right)(x-3)^{2(n+1)}}{(n+1)^2}\cdot\dfrac{n^2}{4^n\left(\tan\frac{2}{n}\right)(x-3)^{2n}}\right|$

$= \boxed{\lim_{n\to\infty}\left|4(x-3)^2\dfrac{n^2}{(n+1)^2}\,\dfrac{\tan\frac{2}{n+1}}{\tan\frac{2}{n}}\right|}$ 6

$\lim_{n\to\infty}\dfrac{n^2}{(n+1)^2} = \boxed{\lim_{n\to\infty}\dfrac{1}{(1+\frac{1}{n})^2}}$ 2 $= \dfrac{1}{(1+0)^2} = 1$

$= \boxed{4(x-3)^2 < 1}$ 3 IF $|2(x-3)| < 1$

$\boxed{-1 < 2(x-3) < 1}$ 3

$-\frac{1}{2} < x-3 < \frac{1}{2}$

$\boxed{\frac{5}{2} < x < \frac{7}{2}}$ 2

SERIES CONV ON $\left(\frac{5}{2},\frac{7}{2}\right)$

$\lim_{n\to\infty}\dfrac{\tan\frac{2}{n+1}}{\tan\frac{2}{n}}\ \left(\frac{0}{0}\right)$

$= \boxed{\lim_{n\to\infty}\dfrac{-\frac{2}{(n+1)^2}\sec^2\frac{2}{n+1}}{-\frac{2}{n^2}\sec^2\frac{2}{n}}}$ 4

$= \lim_{n\to\infty}\dfrac{n^2}{(n+1)^2}\,\dfrac{\sec^2\frac{2}{n+1}}{\sec^2\frac{2}{n}}$

$= 1\cdot\frac{1}{1} = \boxed{1}$ 2

IF $x=\frac{5}{2}$ OR $x=\frac{7}{2}$

$\sum_{n=1}^{\infty}\dfrac{4^n\left(\tan\frac{2}{n}\right)\left(\pm\frac{1}{2}\right)^{2n}}{n^2} = \sum_{n=1}^{\infty}\dfrac{4^n\cdot\left(\frac{1}{4}\right)^n\tan\frac{2}{n}}{n^2} = \boxed{\sum_{n=1}^{\infty}\dfrac{\tan\frac{2}{n}}{n^2}}$ 6

IF $\boxed{x\geq 2}$ 3, $\boxed{0<\frac{2}{x}\leq 1<\frac{\pi}{2}}$ 3, SO $\frac{2}{x}\in\left(0,\frac{\pi}{2}\right)$, SO $\tan\frac{2}{x}>0$ AND $\boxed{\dfrac{\tan\frac{2}{x}}{x^2}>0}$ 2

ALSO $\tan x$ IS CONTINUOUS ON $\left[0,\frac{\pi}{2}\right)$

SO $\dfrac{\tan\frac{2}{x}}{x^2}$ IS $\boxed{\text{CONTINUOUS}}$ 2 ON $[2,\infty)$

SINCE $x^2\neq 0$ AND $\frac{2}{x}\in(0,1)$

$\dfrac{d}{dx}\dfrac{\tan\frac{2}{x}}{x^2} = \boxed{\dfrac{x^2\left(\sec^2\frac{2}{x}\right)\left(-\frac{2}{x^2}\right)-2x\tan\frac{2}{x}}{x^4}}$ 3

$= \boxed{\dfrac{-\left(2\sec^2\frac{2}{x}+2x\tan\frac{2}{x}\right)}{x^4}}$ 3 $\boxed{<0}$ 2 IF $x\geq 2$

SO $\dfrac{\tan\frac{2}{x}}{x^2}$ IS $\boxed{\text{DECREASING}}$ 2 ON $[2,\infty)$

SO INTEGRAL TEST APPLIES

$$\int \frac{\tan\frac{2}{x}}{x^2}\,dx = \boxed{-\frac{1}{2}\int \tan u\,du} = \boxed{\frac{1}{2}\ln\left|\cos\frac{2}{x}\right|} + C$$

3 3

$\boxed{u=\frac{2}{x}}$ 3

$du = -\frac{2}{x^2}\,dx$

$$\int_2^\infty \frac{\tan\frac{2}{x}}{x^2}\,dx = \boxed{\lim_{N\to\infty} \frac{1}{2}\ln\left|\cos\frac{2}{x}\right|\Big|_2^\infty} = \boxed{\lim_{N\to\infty}\frac{1}{2}\left(\ln\left|\cos\frac{2}{N}\right| - \ln|\cos 1|\right)}$$

3 3

$$= \frac{1}{2}\left(\ln|\cos 0| - \ln|\cos 1|\right)$$

$$= \frac{1}{2}\left(\ln 1 - \ln(\cos 1)\right)$$

$$= \boxed{-\frac{1}{2}\ln(\cos 1)}$$ 3

NOTE: THIS IS POSITIVE SINCE $0 < \cos 1 < 1$, so $\ln \cos 1 < 0$

$\boxed{\int_2^\infty \frac{\tan\frac{2}{x}}{x^2}\,dx \text{ CONVERGES}}$ 3

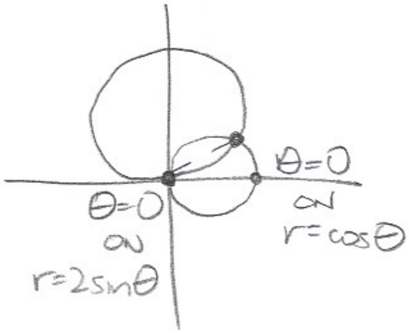
SO $\boxed{\sum_{n=2}^\infty \frac{\tan\frac{2}{n}}{n^2} \text{ CONVERGES (INTEGRAL TEST)}}$ 3

AND $\sum_{n=1}^\infty \frac{\tan\frac{2}{n}}{n^2} = \boxed{\tan 2 + \sum_{n=2}^\infty \frac{\tan\frac{2}{n}}{n^2}}$ $\boxed{\text{CONVERGES}}$ 2

3

SO INTERVAL OF CONVERGENCE IS $\boxed{\left[\frac{5}{2}, \frac{7}{2}\right]}$ 3

[3]



$r=\cos\theta$ $\quad$ $\theta=0 \rightarrow r=1$

$r=2\sin\theta$ $\quad$ $\theta=0 \rightarrow r=0$

FROM LECTURE, BOTH CURVES ARE COMPLETELY DRAWN FOR $\theta \in [0,\pi]$

$\cos\theta=0$, $\theta\in[0,\pi]$

$\theta=\frac{\pi}{2}$ 2

$2\sin\theta=0$, $\theta\in[0,\pi]$

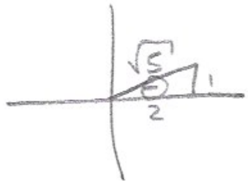
$\sin\theta=0$

$\theta=0,\pi$ 2

$\cos\theta=2\sin\theta$, $\theta\in[0,\pi]$

$\frac{1}{2}=\tan\theta$

$\theta=\tan^{-1}\frac{1}{2}$ 4



$\cos\theta=\frac{2}{\sqrt{5}}$

$\sin\theta=\frac{1}{\sqrt{5}}$

DON'T NEED OTHER INTERSECTIONS FROM SUBSTITUTING (r,θ) WITH $(-r,\pi+\theta)$ BECAUSE $\pi+\theta\notin[0,\pi]$

$$\text{AREA} = \underbrace{\int_0^{\tan^{-1}\frac{1}{2}}}_{4} \underbrace{\frac{1}{2}(2\sin\theta)^2}_{4}\,d\theta + \underbrace{\int_{\tan^{-1}\frac{1}{2}}^{\frac{\pi}{2}}}_{4} \underbrace{\frac{1}{2}(\cos\theta)^2}_{4}\,d\theta$$

$$= \underbrace{2\left(\frac{1}{2}\left(\theta-\frac{1}{2}\sin 2\theta\right)\right)}_{4}\Bigg|_0^{\tan^{-1}\frac{1}{2}} + \underbrace{\frac{1}{2}\left(\frac{1}{2}\left(\theta+\frac{1}{2}\sin 2\theta\right)\right)}_{4}\Bigg|_{\tan^{-1}\frac{1}{2}}^{\frac{\pi}{2}}$$

$$= (\theta-\sin\theta\cos\theta)\Big|_0^{\tan^{-1}\frac{1}{2}} + \frac{1}{4}(\theta+\sin\theta\cos\theta)\Big|_{\tan^{-1}\frac{1}{2}}^{\frac{\pi}{2}}$$

$$= \left(\tan^{-1}\frac{1}{2}-\frac{1}{\sqrt{5}}\frac{2}{\sqrt{5}}\right) + \frac{1}{4}\left(\frac{\pi}{2}-\left(\tan^{-1}\frac{1}{2}+\frac{1}{\sqrt{5}}\frac{2}{\sqrt{5}}\right)\right)$$

$$= \tan^{-1}\frac{1}{2}-\frac{2}{5}+\frac{\pi}{8}-\frac{1}{4}\tan^{-1}\frac{1}{2}-\frac{1}{10}$$

$$= \boxed{\frac{3}{4}\tan^{-1}\frac{1}{2}-\frac{1}{2}+\frac{\pi}{8}} \quad 8$$

[4] $(1-x^2)^{\frac{1}{3}} = 1 + \frac{1}{3}(-x^2) + \frac{\frac{1}{3}\left(-\frac{2}{3}\right)}{2!}(-x^2)^2 + \cdots = \underline{1 - \frac{1}{3}x^2 - \frac{1}{9}x^4 + \cdots}$

4

$\sin 2x = 2x - \frac{(2x)^3}{3!} + \frac{(2x)^5}{5!} + \cdots = \underline{2x - \frac{4}{3}x^3 + \frac{4}{15}x^5 + \cdots}$

4

$$(1-x^2)^{\frac{1}{3}}\sin 2x = \left(1 - \frac{1}{3}x^2 - \frac{1}{9}x^4 + \cdots\right)\left(2x - \frac{4}{3}x^3 + \frac{4}{15}x^5 + \cdots\right)$$

$$= \underline{2x + \left(-\frac{4}{3} - \frac{2}{3}\right)x^3 + \left(\frac{4}{15} + \frac{4}{9} - \frac{2}{9}\right)x^5} + \cdots$$

8

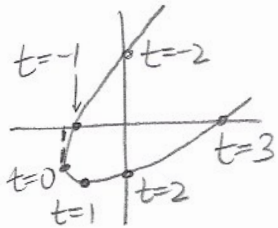
$$= \underline{2x - 2x^3 + \frac{22}{45}x^5 + \cdots}$$

4

[5][a] HORIZONTAL: $\frac{dy}{dt}=0 \longrightarrow 2t-2=0 \longrightarrow t=1 \longrightarrow y=-4$ 4 2

VERTICAL: $\frac{dx}{dt}=0 \longrightarrow 2t=0 \longrightarrow t=0 \longrightarrow x=-4$ 4 2

[b]



$x=0 \longrightarrow t^2-4=0 \longrightarrow t=-2,2$ 4

$y=0 \longrightarrow t^2-2t-3=0 \longrightarrow (t-3)(t+1)=0$

$t=3,-1$ 4

$$\int_{-1}^{-2} y\,dx + -\int_{0}^{2} y\,dx - -\int_{0}^{-1} y\,dx$$ 12

$$= \int_{-1}^{-2} y\,dx + \int_{2}^{0} y\,dx + \int_{0}^{-1} y\,dx$$

$$= \int_{2}^{-2} y\,dx$$

$$= \int_{2}^{-2} (t^2-2t-3)\,2t\,dt$$ 4

$$= \int_{2}^{-2} (2t^3-4t^2-6t)\,dt$$

SYMMETRIC INTERVAL

ODD FUNC

ODD FUNC

$$= \int_{-2}^{2} 4t^2\,dt$$

$$= \frac{4}{3}t^3\Big|_{-2}^{2} = \frac{4}{3}(8--8) = \frac{64}{3}$$ 2 2